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Axions and the Strong CP Problem in M-theory

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Abstract

We examine the possibility that the strong CP problem is solved by string-theoretic axions in strong-coupling limit of the $E_8 \times E_8'$ heterotic string theory (M-theory). We first discuss some generic features of gauge kinetic functions in compactified M-theory, and examine in detail the axion potential induced by the explicit breakings other than the QCD anomaly of the non-linear $U(1)_{PQ}$ symmetries of string-theoretic axions. It is argued based on supersymmetry and discrete gauge symmetries that if the compactification radius is large enough, there can be a $U(1)_{PQ}$-symmetry whose breaking other than the QCD anomaly, whatever its microscopic origin is, is suppressed enough for the axion mechanism to work. Phenomenological viability of such a large radius crucially depends upon the quantized coefficients in gauge kinetic functions. We note that the large radius required for the axion mechanism is viable only in a limited class of models. For instance, for compactifications on a smooth Calabi-Yau manifold with a vanishing E_8' field strength, it is viable only when the quantized flux of the antisymmetric tensor field in M-theory has a minimal nonzero value. It is also stressed that this large compactification radius

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allows the QCD axion in M-theory to be cosmologically viable in the presence of a late time entropy production.

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I. INTRODUCTION AND SUMMARY

It has long been known that $E_8 \times E_8'$ heterotic string theory compactified on a *large* internal six manifold gives rise to an approximate $U(1)_{PQ}$ symmetry which may solve the strong CP problem by means of the axion mechanism [1,2]. Compactified string models always contain the model-independent axion which corresponds to the massless mode of the antisymmetric tensor field in flat spacetime direction. If the internal manifold admits harmonic two forms, which is always the case for the compactification preserving $N = 1$ four-dimensional supersymmetry, the model contains additional model-dependent axions. As is well known, in order for the strong CP problem to be solved by the axion mechanism, one needs a global $U(1)_{PQ}$ symmetry whose *explicit* breaking is almost entirely given by the QCD anomaly $(F\tilde{F})_{QCD}$ [3]. The $U(1)_{PQ}$-breaking other than the QCD anomaly must be so tiny that its contribution to the axion potential satisfies

$$\delta V_{\text{axion}} \lesssim 10^{-9} f_\pi^2 m_\pi^2 \approx 10^{-13}\,\text{GeV}^4. \tag{1}$$

Although string-theoretic axions have the desired coupling to the QCD anomaly, they have potentially harmful additional couplings which would make these axions useless for the strong CP problem. For instance, they generically couple to the hidden sector anomaly $(F\tilde{F})_{\text{hid}}$ [2] and also have non-derivative couplings induced by string world-sheet instantons [4]. As we will discuss later in detail, $U(1)_{PQ}$-breaking by the hidden sector anomaly generically leads to an axion potential much bigger than $10^{-9} f_\pi^2 m_\pi^2$, and then it must be avoided. In fact, in many compactification models, there exists a linear combination of string-theoretic axions which does not couple to the hidden sector anomaly, while keeping the coupling to the QCD anomaly [2]. However such a combination still receives a high energy potential $\delta V_{\text{axion}} \approx e^{-2\pi T} m_{3/2}^2 M_P^2$ from string world-sheet instantons (or equivalently membrane-instantons in M-theory) where T corresponds to the radius-squared of the compact six manifold in the unit of heterotic string tension [4].

If the compactification radius is large enough so that $\text{Re}(T) \gtrsim 20$, world-sheet instanton effects would be small enough to satisfy the bound (1). It has been known that such a large

radius is *not* likely to be allowed within a weakly coupled heterotic string theory [5], and thus its possibility has not been taken seriously. Recently Banks and Dine [6] argued that the large compactification radius giving $\mathrm{Re}(T) \gtrsim 20$ can be realized in strong-coupling limit of the $E_8 \times E_8$ heterotic string theory, i.e. in M-theory limit [7], opening the possibility for the axion solution to the strong CP problem in the context of M-theory. However it has been pointed out subsequently [8] that there is a certain limitation on the compactification radius even in M-theory, which would require a more careful analysis on this problem.

The purpose of this paper is to carefully examine under what conditions the strong CP problem can be solved by string-theoretic axions in M-theory. In the next section, we set up the notations for later analysis by summarizing the known facts on the axions, couplings and scales in heterotic string and M-theories. All couplings and scales are expressed in terms of the dilaton and Kähler modulus superfields, S and T, which makes it convenient to interpret them in the context of four-dimensional effective supergravity model. In sect. III, we discuss some model-independent features of holomorphic gauge kinetic functions in the effective supergravity of heterotic string and M-theories. Particular attention is paid for the quantized coefficients of the Kähler moduli superfields T_I which would determine the phenomenological viability of the axion mechanism in M-theory.

The non-linear $U(1)_{PQ}$ symmetries associated with string-theoretic axions are *explicitly* broken not only by the desired QCD anomaly, but also by the potentially harmful hidden sector gauge anomaly and/or more microscopic stringy (M-theoretic) effects, e.g. the world-sheet (membrane) instantons. In sect. IV, we analyze in detail the high energy potential of the model-independent axion due to the hidden sector anomaly. The resulting axion potential does *not* satisfy the bound (1) *unless* the model is carefully tuned to forbid all dangerous non-renormalizable operators including not only the hidden sector fields but also the observable sector fields. Our study suggests that it is *unlikely* that the strong CP problem is solved by the model-independent axion alone, or at least implementing such a scenario appears to be much more nontrivial than what has appeared in the previous works [9,10].

In sect. V, we consider a $U(1)_{PQ}$-symmetry associated with a linear combination of the

model-independent axion and the model-dependent Kähler axions, which is designed to avoid the hidden sector anomaly. An issue that should be taken account of when one considers the axion solution to the strong CP problem is the possibility of $U(1)_{PQ}$ breaking by generic quantum gravity effects [11]. In this regard, it is desirable that some gauge symmetries protect $U(1)_{PQ}$ from potentially dangerous microscopic stringy (M-theoretic) effects. We argue that supersymmetry and the discrete gauge symmetries highly constrain the explicit breaking of our $U(1)_{PQ}$, and as a result if the compactification radius is large enough to yield $\mathrm{Re}(T) \gtrsim 20$, potentially harmful breaking of $U(1)_{PQ}$ other than the QCD anomaly, *whatever its microscopic origin is*, can be suppressed enough for the axion mechanism to work.

In sect. VI, we discuss the phenomenological viability of such a large radius. It crucially depends upon the quantized coefficients of the Kähler moduli superfields T_I in gauge kinetic functions which can be determined either by the cohomology class of vacuum configuration [12] or by heterotic string one-loop computation [13]. We note that the required large radius is allowed only in a rather limited class of models. For example, for supersymmetry-preserving compactifications on a smooth Calabi-Yau manifold with a vanishing E_8' field strength, it is allowed only when (i) the quantized flux of the antisymmetric tensor field in M-theory has the minimal nonzero value, $[G/2\pi] = 1/2$ in the notation of ref. [14], and (ii) the hidden gauge group E_8' is broken by Wilson lines to a subgroup with small values of the second Casimir $C_2 = \mathrm{tr}(T^2_{\mathrm{adj}})$.

One of the difficulties of solving the strong CP problem by string-theoretic axions would be a too large cosmological axion mass density associated with the axion misalignment $\delta a \gg 10^{12}$ GeV in the early universe [15,2]. Several mechanisms have been suggested to ameliorate this difficulty, for instance an entropy production after the QCD phase transition in the early universe [16,17] or a dynamical relaxation of the axion misalignment [18,19]. In sect. VI, we stress that the large compactification radius allows the QCD axion in M-theory to be cosmologically viable in the presence of a late time entropy production without assuming any significant suppression of the axion misalignment. More explicitly, we find the

axion decay constant $v \approx 10^{16}$ GeV in realistic M-theory limit, for which the QCD axion is cosmologically safe if there is an entropy production with the reheat temperature $T_{RH} \approx 6$ MeV which saturates the lower bound from the big-bang nucleosynthesis. We finally discuss the cosmology of the QCD axion in M-theory in the case that there is an accidental axion-like field whose decay constant is much smaller than 10^{16} GeV. It is pointed out that, due to its high energy potential, such an accidental axion is not so helpful for ameliorating the cosmological difficulty of the original QCD axion in M-theory.

II. AXIONS, COUPLINGS AND SCALES IN M-THEORY

In compactified heterotic string theory, axions appear as the massless modes of the second rank antisymmetric tensor field [1]:

$$B = b_{\mu\nu} dx^\mu dx^\nu + b_I \omega^I_{i\bar{j}} dz^i d\bar{z}^j,$$

where x^μ and z^i denote the coordinates of the flat Minkowski spacetime and the internal six manifold respectively. Here $b_{\mu\nu}$ is the so-called model-independent axion, while b_I are the model-dependent axions associated with the harmonic $(1,1)$ forms $\omega^I_{i\bar{j}}$ $(I = 1, ..., h_{1,1})$ on the compact (complex) six manifold. Such axions remain as massless modes even in strong string coupling limit, i.e. M-theory limit, whose low energy limit can be described by eleven-dimensional supergravity on a manifold with boundary [7]. In this limit, axions appear as the massless modes of the third rank antisymmetric tensor [6]:

$$C = b_{\mu\nu} dx^{11} dx^\mu dx^\nu + b_I \partial_{11} \omega^I_{i\bar{j}} dx^{11} dz^i d\bar{z}^j.$$

In four-dimensional effective supergravity, the model-independent axion $b_{\mu\nu}$ can be identified as the pseudoscalar component of the dilaton superfield S after the duality transformation, while the model-dependent axions b_I correspond to the pseudoscalar components of the Kähler-moduli superfields T_I. To be definite, we normalize S and T_I so that their axion components are periodic fields as:

$$\mathrm{Im}(S) \equiv \mathrm{Im}(S) + 1, \quad \mathrm{Im}(T_I) \equiv \mathrm{Im}(T_I) + 1.$$

This would allow us to define the discrete gauge Peccei-Quinn symmetries as

$$Z_S: \quad S \to S + i, \quad Z_T: \quad T_I \to T_I + i. \tag{2}$$

In this normalization, the world-sheet sigma model action is given by [4]

$$S_{WS} = \frac{1}{4\pi\alpha'} \int d^2z \, [4\pi^2 T_I \omega^I_{i\bar{j}} \partial X^i \bar{\partial} X^{\bar{j}} + 4\pi^2 T_I^* \omega^I_{i\bar{j}} \bar{\partial} X^i \partial X^{\bar{j}}] \tag{3}$$

where the harmonic (1,1) forms ω^I span the integer $(1,1)$ cohomology group of the target space, viz

$$\int_{\Sigma_J} \omega^I = i \int_{\Sigma_J} d^2z \, \omega^I_{i\bar{j}} (\partial X^i \bar{\partial} X^{\bar{j}} - \bar{\partial} X^i \partial X^{\bar{j}}) = 2\alpha' \delta_{IJ}.$$

Note that $S_{WS} \equiv S_{WS} + 2\pi$ correctly leads to the periodicity $T_I \equiv T_I + i$. The Kähler form of the target space is given by $\omega = 4\pi^2 \mathrm{Re}(T_I) \omega^I$, leading to the internal space volume

$$V_6 = \frac{1}{3!} \int \omega \wedge \omega \wedge \omega \approx \frac{1}{6} (4\pi^2 \mathrm{Re}(T))^3 (2\alpha')^3,$$

where the internal six manifold is assumed to be isotropic so that $\mathrm{Re}(T_I) \simeq \mathrm{Re}(T)$.

Using the heterotic string and M-theory relations for the ten-dimensional gauge and gravitational couplings [20,7]

$$\begin{aligned} g_{10}^2 &= e^{2\phi} (2\alpha')^3 = 2\pi (4\pi)^{2/3} l_{11}^6, \\ \kappa_{10}^2 &= \frac{1}{4} e^{2\phi} (2\alpha')^4 = l_{11}^9 / 2R_{11}, \end{aligned}$$

it is now straightforward to find [6]

$$\begin{aligned} \frac{1}{4\pi} \mathrm{Re}(S) &= \frac{1}{g_{GUT}^2} = e^{-2\phi} \frac{V_6}{(2\alpha')^3} = \frac{1}{2\pi (4\pi)^{2/3}} \left(\frac{R_6}{l_{11}} \right)^6, \\ 4\pi^2 \mathrm{Re}(T) &= 6^{1/3} \frac{R_6^2}{2\alpha'} = 6^{1/3} \pi (4\pi)^{2/3} \left(\frac{R_{11}}{l_{11}} \right) \left(\frac{R_6}{l_{11}} \right)^2, \end{aligned} \tag{4}$$

where g_{GUT}, e^ϕ, and $\kappa_{11}^2 = l_{11}^9$ denote the four-dimensional gauge coupling, the heterotic string coupling, and the eleven-dimensional gravitational coupling, respectively, $R_6 = V_6^{1/6}$ is the radius of the internal six manifold, and finally R_{11} is the length of the 11-th interval. ($R_{11} = \pi\rho$ in Witten's notation [21].)

Obvioulsy $\mathrm{Re}(T)$ corresponds to the compactification radius-squared in the heterotic string unit. As we will argue in sect. V, the strong CP problem can be solved by string-theoretic axions if the compactification radius is large enough to yield $\mathrm{Re}(T) \gtrsim 20$. For such a large radius,

$$
\begin{aligned}
& e^{2\phi} \approx g_{GUT}^2 [2\pi^2 \mathrm{Re}(T)]^3 \gtrsim 3 \times 10^7, \\
& R_{11}/R_6 \approx 6^{-1/3} (2\pi)^{-1/2} g_{GUT} \mathrm{Re}(T) \gtrsim 3,
\end{aligned}
\tag{5}
$$

and thus heterotic strings are so strongly coupled that the size of the 11-th dimension is even bigger than that of the other six dimensions.

Using the M-theory expression of the four-dimensional Planck scale [21]

$$
M_P^2 \approx 2 R_{11} V_6 / l_{11}^9 \approx (2.4 \times 10^{18}\,\mathrm{GeV})^2,
$$

the mass scales in compactified M-theory are estimated in terms of g_{GUT} and $\mathrm{Re}(T)$ as:

$$
\begin{aligned}
& \frac{1}{l_{11}} \approx 8 \times 10^{16} \left(\frac{g_{GUT}}{0.7}\right)^{2/3} \left(\frac{20}{\mathrm{Re}(T)}\right)^{1/2} \mathrm{GeV}, \\
& \frac{2\pi}{R_6} \approx 2.5 \times 10^{17} \left(\frac{g_{GUT}}{0.7}\right) \left(\frac{20}{\mathrm{Re}(T)}\right)^{1/2} \mathrm{GeV}, \\
& \frac{2\pi}{R_{11}} \approx 8 \times 10^{16} \left(\frac{20}{\mathrm{Re}(T)}\right)^{3/2} \mathrm{GeV}.
\end{aligned}
\tag{6}
$$

Note that for $R_6 = V_6^{1/6}$ and the length R_{11} of the 11-th interval, the characteristic Kaluza-Klein masses are given in the unit of $2\pi/R_6$ and $2\pi/R_{11}$. It was pointed out in refs. [21,8] that there is a severe limitation on the large radius compactification even in M-theory limit. In ref. [21], it appeared as a lower limit on the Newton's constant G_N for a fixed value of the Kaluza-Klein scale $M_{KK} \approx 2\pi/R_6$, while in ref. [8] it appeared as a lower limit on M_{KK} for a fixed value of the Planck scale $M_P = \sqrt{1/8\pi G_N}$. As we will discuss in sect. VI, in our notation this can be translated into a statement that $\mathrm{Re}(T)$ *can not* be significantly bigger than 20 in order for the four-dimensional gauge coupling constant g_{GUT} to have a realistic value. Thus roughly speaking, what we need for the axion mechanism in M-theory is $\mathrm{Re}(T) \approx 20$. Note that for this value of $\mathrm{Re}(T)$, all typical mass scales in M-theory are

much higher than the dynamical scale of four-dimensional supersymmetry breaking which is for instance given by $\Lambda_{SB} \approx (m_{3/2} M_P^2)^{1/3} \approx 10^{13}$ GeV in non-renormalizable hidden sector models [22] with the weak scale gravitino mass $m_{3/2} \approx 10^2 \sim 10^3$ GeV.

III. GAUGE KINETIC FUNCTIONS IN COMPACTIFIED M-THEORY

In this section, we discuss some model-independent features of holomorphic gauge kinetic functions in the effective four-dimensional supergravity models which correspond to the low energy limit of compactified M-theory. The discrete gauge symmetries $Z_{S,T}$ of Eq. (2) imply that in the limit of $\mathrm{Re}(S) \gg 1$ and $\mathrm{Re}(T_I) \gg 1$, gauge kinetic functions can be written as

$$4\pi f_a = k_a S + \frac{1}{2} l_{aI} T_I + \triangle_a, \tag{7}$$

where k_a and l_{aI} are quantized real coefficients, and $\triangle_a$ denote the piece of order unity (or less) which is independent of S and T_I or the piece which is suppressed by $e^{-2\pi S}$ or $e^{-2\pi T_I}$. Here gauge kinetic functions are normalized as $\mathrm{Re}(f_a) = 1/g_a^2$ and $\mathrm{Im}(f_a) = \theta_a/8\pi^2$ where g_a and θ_a denote the four-dimensional gauge couplings and the Yang-Mills vacuum angles, respectively.

The quantized coefficients k_a and l_{aI} are *unchanged* when one moves from the M-theory domain to the domain of weakly coupled heterotic string in the moduli space of the theory, and thus they can be determined within the weakly coupled heterotic string theory. Note that in the region of $\mathrm{Re}(S) \gg 1$ and $\mathrm{Re}(T_I) \gg 1$ where the expression (7) is valid, the heterotic string can be weakly coupled, $e^{2\phi} \approx 32\pi^7 \mathrm{Re}(T)^3/\mathrm{Re}(S) \ll 1$, in the domain where the four-dimensional gauge couplings are small enough, $g_{GUT}^2 \approx 4\pi/\mathrm{Re}(S) \ll 1/(2\pi^2 \mathrm{Re}(T))^3$. (Of course, $e^{2\phi} \gg 1$ in the domain giving a realistic value of $g_{GUT}^2 \approx \frac{1}{2}$.) In the weak string coupling limit where heterotic string perturbation theory is a good approximation, k_a can be identified (for non-Abelian gauge groups) as the level of the Kac-Moody algebra whose zero modes generate the a-th gauge boson, and thus are positive integers.

If the compactification is simple enough, e.g. orbifolds, one may determine l_{aI} by computing the string one-loop threshold correction to gauge kinetic functions [13]. In fact, one

can extract some model-independent information on l_{aI} without resorting to any string loop calculation. Under the discrete gauge transformation $Z_T : T_I \to T_I + i$, the Yang-Mills vacuum angles transform as

$$\theta_a \to \theta_a + \pi l_{aI},$$

and thus θ_a and $\theta_a + \pi l_{aI}$ are required to be physically equivalent. Applying to the usual 2π periodicity relation $\theta_a \equiv \theta_a + 2\pi$, this may be considered to imply that $l_{aI}/2$ are integers, however this is not necessarily the case in string theory. Due to the existence of the model-independent axion, in string theory the vacuum angles enjoy an additional equivalence relation:

$$\theta_a \equiv \theta_a + 2\pi k_a \gamma, \tag{8}$$

where γ is an arbitrary real constant. We are then led to

$$\frac{1}{2} l_{aI} - k_a \gamma = \text{integer}, \tag{9}$$

and as a consequence

$$\frac{1}{2}(k_b l_{aI} - k_a l_{bI}) = \text{integer}. \tag{10}$$

The equivalence relation (8) can be most easily understood by treating the model-independent axion as an antisymmetric tensor field $b = b_{\mu\nu} dx^\mu \wedge dx^\nu$ whose gauge invariant field strength is given by $H = db - \sum_a k_a \omega^a_{YM} + \omega_L$ where ω^a_{YM} and ω_L denote the Yang-Mills and Lorentz Chern-Simon three forms, respectively [23]. For the gauge invariance of H, only the large gauge transformations under which $\int_{R^3} \sum_a k_a \omega^a_{YM} = \sum_a k_a n_a$ is invariant are allowed in the theory. For such large gauge transformation U, we have $\sum_a k_a (n_a - n'_a) = 0$ where $|n'_a\rangle = U|n_a\rangle$. Then for the θ-vacuum state defined as $|\theta_a\rangle = \sum_{n_a} \exp(i \sum_a n_a \theta_a)|n_a\rangle$, we have the equivalence relation

$$U|\theta_a\rangle = U|\theta_a + k_a \gamma\rangle,$$

and thus the equivalence relation (8) also. In the dual pseudo-scalar description of the model-independent axion which we are using here, the equivalence relation (8) is nothing

but to make one combination of the vacuum angles to be gauged away by the shift of the model-independent axion $\mathrm{Im}(S)$. A property which distinguishes $\mathrm{Im}(S)$ from other axion-like fields is that its non-derivative couplings appear *always* through the combination $\mathrm{Im}(f_a) \propto k_a \mathrm{Im}(S) + \frac{1}{2} l_{aI} \mathrm{Im}(T_I)$ whose vacuum value corresponds to the vacuum angle θ_a. (This amounts to the dual property of the gauge-invariance of $H = db + \omega_L - \sum_a k_a \omega^a_{YM}$ in the antisymmetric tensor formulation.) Due to this special property of $\mathrm{Im}(S)$, vacuum angles related by a shift of $\mathrm{Im}(S)$ are physically equivalent to each other. Model-dependent axions $\mathrm{Im}(T_I)$ have other type of nonderivative couplings in addition to the couplings through $\mathrm{Im}(f_a)$, e.g. those induced by world-sheet instantons and possibly others, and thus do not provide additional equivalence relation for the vacuum angles. However they can still be useful for the strong CP problem if their non-derivative couplings other than the QCD anomaly are suppressed enough.

Let us now focus on the $E_8 \times E'_8$ theory compactified on a large smooth Calabi-Yau manifold and let f_{E_8} and $f_{E'_8}$ denote the visible ($\subset E_8$) and hidden ($\subset E'_8$) sector gauge kinetic functions, respectively. In this case, $k_a = 1$ and thus

$$\begin{aligned} 4\pi f_{E_8} &= S + \frac{l_I}{2} T_I + \triangle_{E_8}, \\ 4\pi f_{E'_8} &= S + \frac{l'_I}{2} T_I + \triangle_{E'_8}. \end{aligned} \tag{11}$$

The coefficients l_I and l'_I can be determined in the limit of $\mathrm{Re}(S) \gg 1$ and $\mathrm{Re}(T) \gg 1$ while $e^{2\phi} \ll 1$, in which ten-dimensional effective field theory provides a good approximation. In this limit, the anomaly cancellation mechanism implies that the axion couplings of $\mathrm{Im}(T_I)$ are entirely due to the Green-Schwarz term in ten-dimensional field theory [24,25,6]:

$$S_{GS} = \frac{1}{288(2\pi)^5} \int B[\mathrm{Tr}(F^4) - \frac{1}{300}(\mathrm{Tr}(F^2))^2 - \frac{1}{10}\mathrm{Tr}(F^2)\mathrm{tr}(R^2) + ...].$$

For the compactification on $M_4 \times M_6$, the above Green-Schwarz term leads to the following axion couplings in the four-dimensional effective theory:

$$\frac{1}{32\pi} \int_{M_6} \omega^I \wedge I_4 \int_{M_4} \mathrm{Im}(T_I)[\mathrm{tr}(F^{\mu\nu}\tilde{F}_{\mu\nu} - F'^{\mu\nu}\tilde{F}'_{\mu\nu})], \tag{12}$$

where $F_{\mu\nu}$ and $F'_{\mu\nu}$ denote the E_8 and E'_8 field strengths on M_4, $\tilde{F}_{\mu\nu}$ and $\tilde{F}'_{\mu\nu}$ are their duals, and the four-form I_4 is defined as

$$8\pi^2 I_4 = \mathrm{tr}(F \wedge F) - \frac{1}{2}\mathrm{tr}(R \wedge R),$$

with $F = \frac{1}{2}F_{AB}dy^A \wedge dy^B$ and $R = \frac{1}{2}R_{AB}dy^A \wedge dy^B$, where y^A correspond to the real coordinates of M_6. Here we set $2\alpha' = 1$, $\mathrm{tr}F^2$ is 1/30 of the trace over the E_8 adjoint representation, and $B = \mathrm{Im}(\tilde{\omega}) = 4\pi^2\mathrm{Im}(T_I)\omega^I$ where $\tilde{\omega}$ denotes the complexified Kähler form. To arrive at the above result, we have used the model-independent constraint

$$\int_{M_6} \omega^I \wedge (I_4 + I'_4) = \frac{1}{8\pi^2}\int_{M_6} \omega^I \wedge dH = 0,$$

where $8\pi^2 I'_4 = [\mathrm{tr}(F' \wedge F') - \frac{1}{2}\mathrm{tr}(R \wedge R)]$. Matching the axion coupling (12) to the gauge kinetic functions in (11), we finally obtain

$$l'_I = -l_I = \int_{M_6} \omega^I \wedge I_4. \tag{13}$$

Note that ω^I are normalized as $\int_{\Sigma_I} \omega_J = \delta_{IJ}$ (in the unit of $2\alpha' = 1$), and then $\frac{1}{2}(l_I - l'_I) = l_I$ are integral as required by (10). The relation $l_I = -l'_I$ was noted before by several authors [24,25,6] for the case of (2,2) Calabi-Yau compactifications. Here we stress that it is valid for generic smooth compactifications and thus is a rather model-independent prediction of string theory. (It has been noted recently that $l_I = -l'_I$ also in some orbifold models [26].)

As was noted by Banks and Dine [6], the weak-coupling result (13) can be confirmed by the Witten's strong-coupling expansion in M-theory [21], which leads to

$$V_6(E'_8) - V_6(E_8) = 2\pi(4\pi)^{-2/3}R_{11}l_{11}^3 \int_{M_6} \omega \wedge I_4,$$

where $V_6(E_8)$ and $V_6(E'_8)$ denote the Calabi-Yau volume on the E_8-boundary and the E'_8-boundary, respectively, and $\omega = 4\pi^2\mathrm{Re}(T_I)\omega^I$ is the Kähler form. With the relation (4), this amounts to

$$4\pi\mathrm{Re}(f'_{E_8}) - 4\pi\mathrm{Re}(f_{E_8}) = \mathrm{Re}(T_I)\int_{M_6} \omega^I \wedge I_4, \tag{14}$$

which gives $\frac{1}{2}(l'_I - l_I) = \int_{M_6} \omega^I \wedge I_4$. The above result indicates also that $\triangle_a$ in the gauge kinetic function corresponds to higher order correction in the strong coupling expansion. Note that, in the M-theory limit under consideration, $\mathrm{Re}(T_I) \gg 1$, while $\mathrm{Re}(\triangle_a)$ are essentially of order unity or less. Also in M-theory, I_4 and I'_4 can be identified as the boundary values of the field strength $G = dC + ...$ of the three form field C in eleven-dimensional supergravity: $\frac{1}{\pi}[G]_{E_8} = I_4$ and $\frac{1}{\pi}[G]_{E'_8} = I'_4$ [7], and then the Bianchi identity $\frac{1}{\pi}\int dG = \int_{E_8} I_4 + \int_{E'_8} I'_4 = 0$ implies $l_I = -l'_I$. We then recover the weak-coupling result (13) through the strong coupling calculation.

For supersymmetry-preserving compactifications on a smooth Calabi-Yau manifold, Eq. (14) has an alternative expression. In both the weakly coupled heterotic string theory and the M-theory limits, the starting point of such compactifications is a manifold of $SU(3)$ holonomy, and a holomorphic Yang-Mills connection satisfying the Kähler-Yang-Mills equations: $F_{ij} = g^{i\bar{j}} F_{i\bar{j}} = 0$. We then have $\int \omega \wedge \mathrm{tr}(R \wedge R) = -\frac{1}{2}\int \mathrm{tr}(R_{AB}R^{AB})$ and similarly $\int \omega \wedge \mathrm{tr}(F \wedge F) = -\frac{1}{2}\int \mathrm{tr}(F_{AB}F^{AB})$. Also the string theory Bianchi identity $dH = 8\pi^2(I_4 + I'_4)$ (or the dilaton equation of motion) or its M-theory counterpart leads to the condition $\int \mathrm{tr}(F_{AB}F^{AB} + F'_{AB}F'^{AB} - R_{AB}R^{AB}) = 0$ [27]. Putting these together, we find

$$4\pi \mathrm{Re}(f_{E_8}) - 4\pi \mathrm{Re}(f_{E'_8}) = \sum l_I \mathrm{Re}(T_I) = \frac{1}{128\pi^4}[\int_{M_6} F_{AB}F^{AB} - \int_{M'_6} F'_{AB}F'^{AB}]. \tag{15}$$

With this expression, it is easy to realize that if the E'_8 field strength does vanish, i.e. $F'_{AB} = 0$, then $\sum l_I \mathrm{Re}(T_I) > 0$ independently of the relative ratios $\mathrm{Re}(T_I)/\mathrm{Re}(T_J)$, and thus at least one of l_I's is a positive integer, while other l_I's are still *non-negative* integers.

IV. HIGH ENERGY POTENTIAL OF THE MODEL-INDEPENDENT AXION

In this and the next sections, we analyze in detail the axion potential due to the explicit breakings other than the QCD anomaly of the non-linear $U(1)_{PQ}$ symmetries of string-theoretic axions. As is well known, in order for the strong CP problem to be solved by the axion mechanism, one needs an anomalous global $U(1)_{PQ}$ symmetry whose breaking other than the QCD anomaly is so tiny that its contribution to the axion potential satisfies

$$\delta V_{\rm axion} \lesssim 10^{-9} f_\pi^2 m_\pi^2 \approx 10^{-13}\,{\rm GeV}^4. \tag{16}$$

To examine the possibility of such a $U(1)_{PQ}$ symmetry, we start from the non-linear $U(1)_S$ symmetry associated with the model-independent axion ${\rm Im}(S)$,

$$U(1)_S : \quad S \rightarrow S + i\alpha,$$

where α is a continuous real parameter. Since $k_a \neq 0$ in both the observable sector and the hidden sector gauge kinetic functions written as (7), $U(1)_S$ is broken not only by the QCD anomaly, but also by the hidden sector gauge anomaly. Our primary concern in this section is to examine the possibility that the high energy potential $\delta V_{{\rm Im}(S)}$ of the model-independent axion due to the hidden sector anomaly is suppressed enough to satisfy the bound (16).

The axion potential $\delta V_{{\rm Im}(S)}$ is somewhat sensitive to how four-dimensional supersymmetry is spontaneously broken. As we will see in sect. VI, ${\rm Re}(T)$ can *not* be significantly bigger than 20, and then the typical mass scales in M-theory estimated in (6) are too large to be identified as the dynamical scale of four-dimensional supersymmetry breaking. It is then the most attractive possibility that the supersymmetry breaking scale is small compared to the typical M-theory scale, say $2\pi R_6^{-1}$ or $2\pi R_{11}^{-1}$ which is about 10^{17} GeV, since it is induced by nonperturbative effects like the hidden sector gaugino (matter) condensations (or some M-theoretical nonperturbative effects). In this scheme, after integrating out the hidden sector gauge and matter multiplets, the resulting effective superpotential can be written as

$$W_{\rm eff} = W_1 + W_2 + W_{\rm obs} = \Omega_1 e^{-2\pi\gamma_1 S} + \Omega_2 e^{-2\pi\gamma_2 S} + W_{\rm obs}, \tag{17}$$

where Ω_1 and Ω_2 depend upon neither S nor the observable sector fields (but generically they depend upon the other moduli), and γ_1 and γ_2 are *different* rational coefficients. Here $W_1 = \Omega_1 e^{-2\pi\gamma_1 S} \approx m_{3/2} M_P^2$ is the leading term in $W_{\rm eff}$, while $W_2 = \Omega_2 e^{-2\pi\gamma_2 S}$ is the next-to-leading term when the observable sector fields are set to zero, and finally $W_{\rm obs}$ includes the terms depending upon the observable sector fields.

Let us first consider the hidden sector contribution to $\delta V_{{\rm Im}(S)}$. From the supergravity potential

$$V_{\rm SG} = e^{K_{\rm eff}/M_P^2}[K_{\rm eff}^{IJ} D_I W_{\rm eff} (D_J W_{\rm eff})^* - 3|W_{\rm eff}|^2/M_P^2] + (\text{D terms}), \tag{18}$$

it is easy to find

$$\delta V_{{\rm Im}(S)} \approx W_1 W_2^*/M_P^2 \approx (W_2^*/W_1) m_{3/2}^2 M_P^2, \tag{19}$$

where $K_{\rm eff}$ is the effective Kähler potential after integrating out the hidden sector gauge and matter multiplets. Obviously, in order for this axion potential to satisfy the bound (16), one needs a huge hierarchy between the leading term W_1 and the next-to-leading term W_2:

$$\frac{W_2}{W_1} \approx \frac{W_2}{m_{3/2} M_P^2} \lesssim 10^{-55} \left(\frac{\rm TeV}{m_{3/2}} \right)^2 .$$

If there are more than one non-Abelian hidden sector gauge groups which would yield multi-gaugino (matter) condensations, W_2 receives a contribution from the second largest condensate. This contribution is typically much bigger than Λ_{QCD}^3. In particular, if the dilaton is stabilized by the racetrack mechanism [28], W_2 and W_1 are comparable to each other, yielding $\delta V_{{\rm Im}(S)} \approx m_{3/2}^2 M_P^2$. We thus conclude that in models with multi-gaugino (matter) condensations, the model-independent axion receives a harmful high energy potential much bigger than $10^{-9} f_\pi^2 m_\pi^2$.

If there is only a single non-Abelian hidden sector gauge group, higher dimensional operators would be responsible for the next-to-leading term W_2 in the effective superpotential (17). (One may suffer from the dilaton runaway in this case. Here we assume as in ref. [9] that the dilaton is stabilized by a large $U(1)_S$-preserving nonperturbative correction to the Kähler potential.) As an example, let us consider a typical hidden sector with $SU(N_c)$ gauge group, N_f quark flavors $(Q + Q^c)$ and N_s singlets A, and the tree level superpotential

$$W_{\rm tree} = A^3 + AQQ^c + ...,$$

where the Yukawa couplings of order unity are omitted and the ellipsis denotes higher dimensional operators. We also assume $k_a = 1$ for the hidden sector gauge kinetic function written as (7). We then have

$$W_1 \approx \langle W_{\rm hid}^a W_{\rm hid}^a \rangle \approx \Lambda_{\rm hid}^3, \quad \langle A \rangle \approx \Lambda_{\rm hid}, \quad \langle QQ^c \rangle \approx \Lambda_{\rm hid}^2,$$

where $W^a_{\rm hid}$ is the chiral superfield whose lowest component corresponds to the hidden sector gaugino, and $\Lambda_{\rm hid} \propto e^{-2\pi S/(3N_c-N_f)}$ is the dynamical scale of the hidden sector gauge interaction. In this case, the next-to-leading term W_2 can be induced by gauge-invariant non-renormalizable operators of the form

$$M_P^3 \int d^2\theta \left(\frac{W^a_{\rm hid}W^a_{\rm hid}}{M_P^3}\right)^n \prod_{k=1}^{m}\left(\frac{\Phi_{I_k}}{M_P}\right), \tag{20}$$

where Φ_I denote the hidden matter superfields: $\Phi_I = (A, Q, Q^c)$. Obviously the above operator gives rise to

$$W_2 \approx \frac{\Lambda_{\rm hid}^{3n+m}}{M_P^{3n+m-3}} \approx M_P^3\left(\frac{W_1}{M_P^3}\right)^{(3n+m)/3}.$$

and thus

$$\delta V_{{\rm Im}(S)} \approx \frac{W_1 W_2^*}{M_P^2} \approx m_{3/2}^2 M_P^2 \left(\frac{m_{3/2}}{M_P}\right)^{(3n+m-3)/3}. \tag{21}$$

In order for this axion potential to satisfy the bound (16), we need

$$3(n-1)+m \gtrsim \frac{380+6\ln(m_{3/2}/{\rm TeV})}{35-\ln(m_{3/2}/{\rm TeV})}, \tag{22}$$

i.e. all non-renormalizable operators whose mass dimension $d = 3n+m+1 \lesssim 14$ have to be forbidden.

It has been pointed out in [9] that, if there is no hidden matter, a simple discrete gauge symmetry can eliminate all dangerous non-renormalizable operators. The example considered in [9] is a discrete gauge R symmetry Z_5 under which $d^2\theta \to e^{-i2\pi/5}d^2\theta$. The model-independent axion is transformed also as $S \to S + iN_c/5$ to cancel the $Z_5 \times SU(N_c) \times SU(N_c)$ anomaly. In the absence of any hidden matter multiplet, this Z_5 allows only the operators with $n \geq 6$ in (20), thereby satisfying the condition (22). However, if the hidden sector contains matter multiplets, it becomes much more nontrivial to fulfill the condition (22). The Z_5-charges of the matter multiplets have to be judiciously chosen in order to forbid all dangerous non-renormalizable operators with the mass dimension $d \lesssim 14$.

Even when the hidden sector is adjusted to make its contribution to $\delta V_{{\rm Im}(S)}$ satisfy the bound (16), one still has to carefully tune the observable sector to avoid a harmful axion

potential from the observable sector dynamics. Let us suppose that the hidden sector is tuned to have $W_2/W_1 \lesssim 10^{-55}$ by means of a discrete gauge symmetry, e.g. Z_5 of ref. [9]. To see how a sizable $\delta V_{\mathrm{Im}(S)}$ can still be induced, let us consider a model whose observable sector contains a matter multiplet ϕ which is neutral under unbroken continuous gauge symmetries, but carries a nonzero Z_5-charge q_ϕ:

$$Z_5: \quad d^2\theta \to e^{-i2\pi/5} d^2\theta, \quad \phi \to e^{i2\pi q_\phi/5}\phi.$$

Note that such a matter multiplet appears quite often in compactified string models. Although ϕ does not have any renormalizable coupling with the hidden sector fields, it can still couple to the hidden sector via non-renormalizable operators. For instance, gauge symmetries including Z_5 allow the couplings of the form

$$M_P^3 \int d^2\theta \left[\left(\frac{W^a_{\rm hid} W^a_{\rm hid}}{M_P^3} \right)^N \left(\frac{\phi}{M_P} \right)^M + \left(\frac{\phi}{M_P} \right)^L \right], \tag{23}$$

and thus the folllowing effective superpotential

$$W_{\rm eff} = \Omega_1 e^{-2\pi\gamma_1 S} + M_P^{3(1-N)} \Omega_1^N e^{-2\pi N \gamma_1 S} \left(\frac{\phi}{M_P} \right)^M + \left(\frac{\phi}{M_P} \right)^L .$$

for appropriate values of the positive integers N, M, and L. To be more specific, let us consider the case (I) with $q_\phi = -1$ for which $N = 1$, $M = 5$, $L = 4$, and also the case (II) with $q_\phi = 2$ for which $N = 2$, $M = 2$, $L = 3$. Then the supergravity potential (18) contains

$$\delta V_{SG} \approx \begin{cases} (m^*_{3/2}\phi^4/M_P) + (|m_{3/2}|^2\phi^5/M_P^3) & : \text{Case (I)} \\ (m^*_{3/2}\phi^3) + (|m_{3/2}|^2 m_{3/2}\phi^2/M_P) & : \text{Case (II)} \end{cases}$$

where the model-independent axion $\mathrm{Im}(S)$ appears through the complex gravitino mass defined as $m_{3/2} \approx W_1/M_P^2 = \Omega_1 e^{-2\pi\gamma_1 S}/M_P^2$. Although it appears to depend upon $\mathrm{Im}(S)$, the first term of the above supergravity potential in each case does *not* contribute to the true axion potential since their $\mathrm{Im}(S)$-dependence can be eliminated by the field redefinitions: $\phi \to e^{-i\pi\gamma_1 \mathrm{Im}(S)/2}\phi$ for the case (I) and $\phi \to e^{-i2\pi\gamma_1 \mathrm{Im}(S)/3}\phi$ for the case (II). As a result, the high energy potential of the model-independent axion in each case is estimated to be

$$
\begin{aligned}
&\text{Case (I)}: \quad \delta V_{\mathrm{Im}(S)} \approx |m_{3/2}|^2 \langle\phi\rangle^5 / M_P^3, \\
&\text{Case (II)}: \quad \delta V_{\mathrm{Im}(S)} \approx |m_{3/2}|^2 m_{3/2} \langle\phi\rangle^2 / M_P.
\end{aligned} \tag{24}
$$

Of course it crucially depends upon the size of $\langle\phi\rangle$ whether the above axion potential satisfies the bound (16). In fact, a nonzero $\langle\phi\rangle$ can lead to interesting phenomenological consequences. For instance, it may generate the μ-term of the Higgs doublets H_u and H_d through the coupling $\int d^2\,\theta\phi H_u H_d$ with the weak scale vacuum value $\langle\phi\rangle \approx m_{3/2}$, or through the coupling $\frac{1}{M_P}\int d^2\theta\,\phi^2 H_u H_d$ with the intermediate scale vacuum value $\langle\phi\rangle = \sqrt{m_{3/2} M_P}$. It may also generate an intermediate scale mass of the right-handed neutrino N through the coupling $\int d^2\theta\,\phi NN$, leading to the small neutrino mass $m_\nu \approx m_{3/2}^2/\sqrt{m_{3/2} M_P}$ via the seesaw mechanism. In case (I), ϕ has a flat potential in the limit of $m_{3/2} \to 0$ and $M_P \to \infty$. Including the non-renormalizable but supersymmetric potential term $|\phi|^6/M_P^2$ together with the soft mass term $m_{3/2}^2|\phi|^2$ which receives a negative radiative correction due to the Yukawa coupling $\int d^2\theta\,\phi NN$, one easily finds $\langle\phi\rangle \approx \sqrt{m_{3/2} M_P}$ for the case (I) [29]. Also for the case (II) with the Yukawa coupling $\int d^2\theta\,\phi H_u H_d$, a similar analysis yields $\langle\phi\rangle \approx m_{3/2}$. Then in both cases (I) and (II), the high energy potential of the model-independent axion $\delta V_{\mathrm{Im}(S)} \gg 10^{-9} f_\pi^2 m_\pi^2$ for the weak scale gravitino mass $m_{3/2} \approx 10^2 \sim 10^3$ GeV.

In fact, $\delta V_{\mathrm{Im}(S)}$ is naively expected to be of order $m_{3/2}^2 M_P^2$ since it is essentially due to hidden sector dynamics triggering supersymmetry breaking. As we have noted, it is really of order $m_{3/2}^2 M_P^2$ in generic cases with multi-gaugino (matter) condensations whose sizes are comparable to each other. In the cases that $\delta V_{\mathrm{Im}(S)} \ll m_{3/2}^2 M_P^2$, there is a simple explanation for a small $\delta V_{\mathrm{Im}(S)}$. In such cases, we always have an approximate *accidental* global $U(1)_X$ symmetry whose current divergence is given by $\partial_\mu J_X^\mu = (F\tilde{F})_1 + \Gamma_X$ where $(F\tilde{F})_1$ corresponds to the $U(1)_X \times G_1 \times G_1$ anomaly for the non-Abelian gauge group G_1 which is responsible for the leading term W_1 in the effective superpotential (17), and Γ_X stands for the other symmetry breaking terms which are presumed to be weaker than the

strongest anomaly $(F\tilde{F})_1$. For the models that we have discussed above, we have

$$U(1)_X: \quad \Phi \to e^{i\beta X(\Phi)}\Phi,$$

where the $U(1)_X$ charges are given by $X(d^2\theta) = -1$, $X(A) = 1/3$, $X(QQ^c) = 2/3$, and $X(\phi) = 1/4$ $(1/3)$ for the case (I) (the case (II)), and then Γ_X includes the $U(1)_X$-breakings by the non-renormalizable operators in (20) and (23). For $k_a = 1$, the model-independent axion has a universal coupling to the gauge anomalies and then the $U(1)_S$ current $J^\mu_S \propto \partial^\mu \mathrm{Im}(S)$ obeys $\partial_\mu J^\mu_S = \sum_a (F\tilde{F})_a + (R\tilde{R})$ where $(R\tilde{R})$ denotes the irrelevant gravitational chiral anomaly. In the presence of $U(1)_X$, the model-independent axion can be identified as the pseudo-Goldstone boson of

$$U(1)_{S-X}: \quad S \to S + ik\beta, \quad \Phi \to e^{i\beta X(\Phi)}\Phi, \tag{25}$$

where the coefficient k is chosen to make $U(1)_{S-X}$ to be free from the strongest hidden sector anomaly $(F\tilde{F})_1$. If the hidden sector gauge group is semisimple, $U(1)_{S-X}$ is still broken by the second strongest hidden sector anomaly, leading to $\delta V_{\mathrm{Im}(S)} \approx W_1 W_2^*/M_P^2 \gg 10^{-9} f_\pi^2 m_\pi^2$ where W_2 is induced by the second strongest hidden sector gauge interaction. In the case that G_1 is the only non-Abelian hidden sector gauge group, major explicit breaking of $U(1)_{S-X}$ is due to the non-renormalizable operators (20) and (23). The axion potential $\delta V_{\mathrm{Im}(S)}$ is then suppressed by an insertion of these small $U(1)_{S-X}$-breaking operators as can be seen in (21) and (24). Although the suppression was not enough so that $\delta V_{\mathrm{Im}(S)} \gg 10^{-9} f_\pi^2 m_\pi^2$ in our examples, there may exist a compactified string model with an accidental $U(1)_X$ which is good enough to yield $\delta V_{\mathrm{Im}(S)} \lesssim 10^{-9} f_\pi^2 m_\pi^2$. However this possibility is too much model-dependent and implementing this scenario in the context of string/M-theory appears to be quite nontrivial.

In summary, in this section we have examined the possibility that the high energy potential $\delta V_{\mathrm{Im}(S)}$ of the model-independent axion due to the hidden sector anomaly is suppressed enough to be smaller than $10^{-9} f_\pi^2 m_\pi^2$, thereby solving the strong CP problem by the model-independent axion alone. First of all, this appears to be *not* possible if the hidden sector

dynamics yields multi-gaugino (matter) condensates. Even in the case that there is only one non-Abelian hidden sector gauge group, it requires a careful tuning of both the hidden sector and the observable sector to forbid all dangerous higher dimensional operators. This is equivalent to having an accidental global $U(1)_X$ for which the combination $U(1)_{S-X}$ of Eq. (25) which is designed to be free from the hidden sector anomaly is so good a symmetry that the corresponding high energy axion potential is smaller than $10^{-9} f_\pi^2 m_\pi^2$. Arranging the observable sector to have such $U(1)_X$ appears to be highly nontrivial. At any rate, our study in this section shows that it is much more nontrivial than what has been suggested in the previous works [9,10] to solve the strong CP problem by the model-independent axion alone.

V. PECCEI-QUINN SYMMETRY IN THE LARGE RADIUS LIMIT

In M-theory limit where the world-sheet (membrane) instanton effects are highly suppressed, the desired $U(1)_{PQ}$ symmetry satisfying the bound (1) may appear as a linear combination of the nonlinear $U(1)$ symmetries of the model-independent axion and the model-dependent Kähler axions [6]. To examine this possibility, let us consider a model compactified on a smooth Calabi-Yau manifold with $h_{1,1} = 1$, and also assume that there is only one non-Abelian hidden sector gauge group from E_8'. It is rather straightforward to generalize our discussion to more general cases that $h_{1,1} > 1$ and/or the hidden sector gauge group is semi-simple. Our starting point is the visible and hidden sector gauge kinetic functions f_{E_8} and $f_{E_8'}$ in the limit of $\mathrm{Re}(S) \gg 1$ and $\mathrm{Re}(T) \gg 1$:

$$
\begin{aligned}
4\pi f_{E_8} &= S + \frac{l}{2}T + \triangle_{E_8}, \\
4\pi f_{E_8'} &= S - \frac{l}{2}T + \triangle_{E_8'},
\end{aligned}
\tag{26}
$$

where l is integral, and again $\triangle_a$ corresponds to the piece of order one which is independent of S and T or the piece which is suppressed by $e^{-2\pi S}$ or $e^{-2\pi T}$. The above form of gauge kinetic functions naturally leads to the following $U(1)_{PQ}$ symmetry

$$U(1)_{PQ}: \quad S \to S + i\alpha, \quad T \to T + 2i\alpha/l,, \tag{27}$$

which is free from the hidden sector anomaly. This $U(1)_{PQ}$ would solve the strong CP problem if its explicit breaking other than the QCD anomaly is so tiny that the associated axion potential satisfies the bound (1). Note that l is required to be nonzero in order to avoid the hidden sector anomaly, while keeping the breaking by the QCD anomaly.

It has been argued that generic quantum gravity effects may break $U(1)_{PQ}$ explicitly [11]. Although it is somewhat clear that world-sheet (membrane) instanton effects are suppressed by $e^{-2\pi T}$ in the large radius limit, in the absence of the full understanding of the M-theory dynamics, one may still wonder that some unknown M-theoretical effects other than world-sheet (membrane) instantons breaks $U(1)_{PQ}$ significantly even in the large radius limit. In this regard, it would be desirable if $U(1)_{PQ}$ in the large radius limit can be protected by some gauge symmetries at the compactification scale. In the following, we argue that supersymmetry and the discrete gauge symmetries highly constrain the possible explicit breaking of $U(1)_{PQ}$, and as a result the potentially harmful breaking of $U(1)_{PQ}$, whatever its microscopic origin is, is suppressed enough if the compactification radius is large enough.

To proceed, let us assume that the discrete gauge symmetries $Z_{S,T}$ of Eq. (2) are *not* spontaneously broken by the M-theory dynamics at scales above the compactification scale, and consider the limit

$$\mathrm{Re}(T) \gg 1, \quad \mathrm{Re}(S) - \frac{l}{2}\mathrm{Re}(T) \gg 1,$$

in which the four-dimensional gauge couplings $g_a^2 = \mathrm{Re}(f_a)^{-1}$ at the compactification scale are small enough. In this limit, $U(1)_{PQ}$-breaking in a holomorphic operator F which is generated by the $Z_{S,T}$-preserving M-theory dynamics is estimated as:

$$\delta_{PQ}F \approx M_P^d \exp[-2\pi\{pT + q(S - lT/2)\}], \tag{28}$$

where p is a positive integer, while q is a non-negative integer. Here d denotes the mass dimension of F and we have used the fact that all non-derivative couplings of $\mathrm{Im}(S)$ are required to appear through the combinations $\mathrm{Im}(f_a) \propto \mathrm{Im}(S) \pm \frac{1}{2}l\mathrm{Im}(T)$. Note that in order

to be a $U(1)_{PQ}$-breaking piece, p is required to be non-zero, while the coefficient q of the $U(1)_{PQ}$-invariant combination $(S - \frac{1}{2}lT)$ can be zero.

Again the integers p and q in (28) are unchanged when one moves from the M-theory domain to the domain of weakly coupled heterotic string, and thus they can be determined within the weakly coupled heterotic string theory. For holomorphic gauge kinetic functions, $U(1)_{PQ}$ is generically broken by world-sheet instanton effects without any suppression by $e^{-2\pi S}$ at string one-loop order [13], and thus

$$\delta_{PQ} f_{E_8} \approx \delta_{PQ} f_{E_8'} \approx e^{-2\pi T}, \tag{29}$$

where δ_{PQ} means $U(1)_{PQ}$-breaking other than the QCD anomaly. Let

$$W = W_M(S,T) + \sum \frac{1}{M_P^n} \lambda_n(S,T) \Phi^{3+n}$$

denote the superpotential at the compactification scale where Φ represents generic chiral matter multiplets (including those in the hidden sector) with the Yukawa-type couplings $\lambda_n(S,T)$. If $W_M = 0$ at string tree level, which is the case in many interesting models including $(2,2)$ Calabi-Yau and orbifold compactifications, it remains to be zero at any finite order in string perturbation theory. (For $(2,0)$ Calabi-Yau compactifications, a nonzero W_M may be induced by world-sheet instantons even at string tree level [4].) However non-perturbative stringy effects may generate a nonzero $W_M \approx M_P^3 e^{-2\pi(S-lT/2)}$, and then $\delta_{PQ} W_M \approx M_P^3 e^{-2\pi[T+(S-lT/2)]}$. It is known that world-sheet instantons can induce $U(1)_{PQ}$-breaking Yukawa-type couplings at string tree level without any suppression by $e^{-2\pi S}$ [4]. Summarizing these, if $W_M = 0$ at string tree level in the weak coupling limit, which is the case that we focus on here, the following order of magnitude estimate applies for $U(1)_{PQ}$-breaking in the superpotential:

$$\begin{aligned} &\delta_{PQ} W_M \approx M_P^3 e^{-2\pi[T+(S-lT/2)]} \approx e^{-2\pi T} W_M, \\ &\delta_{PQ} \lambda_n \approx e^{-2\pi T}. \end{aligned} \tag{30}$$

The discussion of $U(1)_{PQ}$-breaking in the Kähler potential is more subtle because of the absence of holomorphy. Again together with the property that nonderivative couplings of

$\mathrm{Im}(S)$ appear always through the combinations $\mathrm{Im}(f_a) \propto \mathrm{Im}(S \pm \frac{1}{2}lT)$, the discrete symmetries $Z_{S,T}$ imply that generic Kähler potential can be written as $K = \sum_n K_n \exp[i2\pi n \mathrm{Im}(T)]$ where K_n is a function of $\mathrm{Re}(S)$, $\mathrm{Re}(T)$, $\mathrm{Im}(S - \frac{1}{2}lT)$, and also of other $U(1)_{PQ}$-invariant field variables. Obviously $U(1)_{PQ}$ is broken only by K_n with $n \neq 0$. To estimate its size, one may consider the limit of weakly coupled heterotic string which preserves four-dimensional $N = 2$ supersymmetry or the M-theory limit in which the eleventh radius $R_{11} \gg R_6$ so that physics below the energy scale $E \lesssim R_6^{-1}$ can be described by a five-dimensional supergravity. In these limits, T corresponds to the coordinate of a special Kähler manifold whose Kähler potential is determined by a holomorphic prepotential $\mathcal{F}$ as $K = -\ln[2(\mathcal{F} + \mathcal{F}^*) - (\phi_i + \phi_i^*)(\partial_i \mathcal{F} + \partial_i \mathcal{F}^*)]$ [30], and as a result $K_n \propto e^{-2\pi n T}$ as in the case of the holomorphic gauge kinetic functions and superpotential. As long as $Z_{S,T}$ are not spontaneously broken, turning on the spontaneous breaking of four-dimensional $N = 2$ supersymmetry or of five-dimensional supersymmetry down to four-dimensional $N = 1$ supersymmetry does not affect this behavior of $U(1)_{PQ}$-breaking in the large radius limit, and thus

$$\delta_{PQ} K \approx M_P^2 e^{-2\pi T}, \tag{31}$$

in generic large radius compactifications preserving four-dimensional $N = 1$ supersymmetry.

In the above, we have noted that supersymmetry and the discrete gauge symmetries $Z_{S,T}$ associated with the periodicity of the axion-like fields $\mathrm{Im}(S)$ and $\mathrm{Im}(T)$ imply that $U(1)_{PQ}$-breaking terms (other than the QCD anomaly) are suppressed by $e^{-2\pi T}$ in the large radius limit. Although it is quite reasonable to assume that the discrete symmetries $Z_{S,T}$ are *not* spontaneously broken by the M-theory dynamics above the compactification scale, they may be broken by infrared dynamics at scales below the compactification scale. This does indeed occur if non-perturbative hidden sector dynamics leads to the formation of the gaugino and/or matter condensations.

Integrating out the hidden sector gauge and matter multiplets leads to an effective supergravity model of the visible sector fields and also generic moduli including S and T. This effective supergravity will be described by the effective Kähler potential $K_{\rm eff}$, the effective

visible sector gauge kinetic function $f_{\rm eff}$, and finally the effective superpotential $W_{\rm eff}$ which can be written as

$$W_{\rm eff} = W_0(S,T) + \mu(S,T)\Phi^2 + \sum \frac{1}{M_P^n} h_n(S,T)\Phi^{n+3}, \tag{32}$$

where now Φ stands for the visible sector matter multiplets. Even for $K_{\rm eff}$, $f_{\rm eff}$, and $W_{\rm eff}$ including nonperturbative corrections due to the field-theoretic hidden sector dynamics, it is rather obvious that

$$\delta K_{\rm eff} \approx M_P^2 e^{-2\pi T}, \quad \delta_{PQ} f_{\rm eff} \approx e^{-2\pi T}, \qquad \delta_{PQ} h_n \approx e^{-2\pi T}. \tag{33}$$

However one needs a further discussion to estimate $\delta_{PQ}W_0$ and $\delta_{PQ}\mu$. To proceed, let us split W_0 into the two pieces as

$$W_0 = W_M + W_F, \tag{34}$$

where W_M is the piece generated by $Z_{S,T}$-preserving M-theory dynamics at scales above the compactification scale, while W_F (and also μ in (32)) is the piece from field-theoretic infrared effects, i.e. the gaugino and/or matter condensations, which break $Z_{S,T}$ spontaneously. Generically W_F and μ are holomorphic functions of the hidden sector Yukawa couplings λ_n and the dynamical scale $\Lambda_{E_8'} = M_{GUT}\exp[-8\pi^2 f_{E_8'}/b]$ of the hidden sector gauge interaction. (Here b is the coefficient of the one-loop beta function and M_{GUT} can be identified as $2\pi R_6^{-1}$ in (6).) More explicitly,

$$\begin{aligned} W_F &\approx \kappa_1 M_{GUT}^3 (\Lambda_{E_8'}/M_{GUT})^{n_1/n_2} \approx M_{GUT}^3 \exp[-8\pi^2 n_1 f_{E_8'}/n_2 b], \\ \mu &\approx \kappa_2 M_{GUT} (\Lambda_{E_8'}/M_{GUT})^{n_3/n_4} \approx M_{GUT} \exp[-8\pi^2 n_3 f_{E_8'}/n_4 b], \end{aligned} \tag{35}$$

where κ_1 and κ_2 are dimensionless functions of λ_n, and n_i's are model-dependent positive integers. If there is no hidden matter or if the hidden matter multiplets have renormalizable Yukawa couplings, $W_F \approx \Lambda_{E_8'}^3$ and thus $n_1/n_2 = 3$. However if the hidden matters have only non-renormalizable Yukawa-type couplings, n_1/n_2 can take a different value. Note that if $n_1/n_2 b$ or $n_3/n_4 b$ is not integral, which is usually the case, $Z_S : S \to S + i$ is spontaneously

broken with the multi-valued superpotential: $W_F \propto e^{-2\pi n_1 S/n_2 b}$ and $\mu \propto e^{-2\pi n_3 S/n_4 b}$. At any rate, (29) and (30) now imply

$$\begin{aligned} \delta_{PQ} W_F &= \frac{\partial W_F}{\partial f_{E_8'}} \delta_{PQ} f_{E_8'} + \frac{\partial W_F}{\partial \lambda_n} \delta_{PQ} \lambda_n \approx W_F e^{-2\pi T}, \\ \delta_{PQ} \mu &= \frac{\partial \mu}{\partial f_{E_8'}} \delta_{PQ} f_{E_8'} + \frac{\partial \mu}{\partial \lambda_n} \delta_{PQ} \lambda_n \approx \mu e^{-2\pi T}. \end{aligned} \tag{36}$$

The axion potential due to $U(1)_{PQ}$-breaking other than the QCD anomaly can be schematically written as

$$\delta V_{\rm axion} = \frac{\delta V_{\rm axion}}{\delta K_{\rm eff}} \delta_{PQ} K_{\rm eff} + \frac{\delta V_{\rm axion}}{\delta W_{\rm eff}} \delta_{PQ} W_{\rm eff} + \frac{\delta V_{\rm axion}}{\delta f_{\rm eff}} \delta_{PQ} f_{\rm eff}.$$

Carefully inspecting the supergravity potential (18) and also all possible quantum corrections including the quadratically divergent one-loop potential $V_{\rm loop} = \frac{1}{16\pi^2} {\rm Str}(M^2)\Lambda^2$ with the cutoff $\Lambda \approx M_{GUT}$ or M_P, it is not difficult to see that

$$\frac{\delta V_{\rm axion}}{\delta K_{\rm eff}} \approx m_{3/2}^2, \quad \frac{\delta V_{\rm axion}}{\delta W_{\rm eff}} \approx m_{3/2}, \quad \frac{\delta V_{\rm axion}}{\delta f_{\rm eff}} \approx \frac{1}{16\pi^2} m_{3/2}^2 \Lambda^2,$$

where the main contributions to $\delta V_{\rm axion}/\delta K_{\rm eff}$ and $\delta V_{\rm axion}/\delta W_{\rm eff}$ are from the tree level supergravity potential (18) with the relation $W_{\rm eff} \approx W_0 \approx m_{3/2} M_P^2$, while $\delta V_{\rm axion}/\delta f_{\rm eff}$ is mainly from $V_{\rm loop}$ through the gaugino masses which depends upon the gauge coupling, i.e. upon ${\rm Re}(f_{\rm eff})$. (Here we assume that supersymmetry breaking is mainly due to the F-terms, not by the D-terms.) Note that $U(1)_{PQ}$-breaking in $W_{\rm eff}$ is dominated by $\delta_{PQ} W_0 = \delta_{PQ}(W_M + W_F)$ which is of order $e^{-2\pi T} W_0 \approx e^{-2\pi T} m_{3/2} M_P^2$ for $\delta_{PQ} W_M$ and $\delta_{PQ} W_F$ estimated in (30) and (36), respectively. Putting these together with the previously made estimates of $U(1)_{PQ}$-breaking, what we find is a rather simple result:

$$\delta V_{\rm axion} \approx m_{3/2}^2 M_P^2 e^{-2\pi T}. \tag{37}$$

With this, we conclude that if the compactification radius is large enough to yield

$${\rm Re}(T) \gtrsim 20 + \frac{1}{\pi} \ln(m_{3/2}/{\rm TeV}), \tag{38}$$

$U(1)_{PQ}$ breaking other than the QCD anomaly, *whatever its microscopic origin is*, is suppressed enough to satisfy the condition (1) for the strong CP problem to be solved by the axion mechanism.

As we have noted, it is more precise to interprete our estimates of $U(1)_{PQ}$-breakings as an approximate upper limit. One might then interprete the estimate of the high energy axion potential in (37) also as an upper limit. However as long as anyone of $\delta_{PQ}K$, $\delta_{PQ}f_{E_8'}$, and $\delta_{PQ}\lambda$ saturates their estimated upper limits, which is true for the most of known compactified string models, (37) corresponds to a correct order of magnitude estimate, not merely an upper limit. (Here λ corresponds to the lowest dimensional non-vanishing Yukawa coupling of hidden matter.) If only $\delta_{PQ}f_{E_8}$ or $\delta_{PQ}h_0$ saturates the bound, the resulting high energy axion potential would be of order of $m_{3/2}^2\Lambda^2/(16\pi^2)^k$ where $k=1$ for f_{E_8} and $k=2$ for the visible matter Yukawa coupling h_0.

Our argument in this section can be easily generalized to the cases with a semisimple hidden sector gauge group and/or the number of the model-dependent Kähler axions $h_{1,1} > 1$. Note that one may need a semisimple hidden sector gauge group with nontrivial matter contents in order to stabilize the dilaton and moduli vacuum expectation values through the racetrack mechanism [28]. For the gauge kinetic functions written as (7), one can define a $U(1)_{PQ}$-symmetry similarly as (17), e.g.

$$U(1)_{PQ}: \quad S \rightarrow S + i\alpha, \quad T_I \rightarrow T_I + ik_I\alpha,$$

where the real coefficients k_I are chosen to make this $U(1)_{PQ}$ to be free from any of the hidden sector anomalies, while be broken by the QCD anomaly. Such a $U(1)_{PQ}$ exists always if $h_{1,1} \geq N_H$ where N_H denotes the number of simple gauge groups in hidden sector. It would exist even when $h_{1,1} < N_H$ if some of the hidden sector gauge kinetic functions are not linearly-independent from each other. Then $U(1)_{PQ}$-breaking other than the QCD anomaly will be suppressed enough if the compactification radius is large enough so that all T_I with $\delta_{PQ}T_I \neq 0$ satisfy the condition (38).

VI. PHENOMENOLOGICAL CONSTRAINTS AND AXION COSMOLOGY

In the previous section, we have argued that if the compactification radius is large enough so that

$$\mathrm{Re}(T_I) \gtrsim 20 + \frac{1}{\pi}\ln(m_{3/2}/\mathrm{TeV}) \tag{39}$$

for all T_I with $\delta_{PQ}T_I \neq 0$, the strong CP problem can be solved by string-theoretic axions. Perhaps the most serious difficulty with this large radius compactification would be to stabilize the dilaton and moduli at the desired vacuum values. Here we simply assume that the dilaton and moduli can be stabilized at a point satisfying (39), and just look at its phenomenological viability.

For the gauge kinetic functions written as (7), most of the known heterotic string models (and thus their M-theory limits also) give $k_a = 1$ [31]. Let f_{E_8} of Eq. (11) denote the QCD gauge kinetic function and $f_{E_8'}$ denote the gauge kinetic function for the hidden sector gauge interaction which gives a dominant contribution to the field-theoretic nonperturbative superpotential W_F in (34). Then the phenomenological value of α_{QCD} at M_{GUT} gives

$$4\pi\mathrm{Re}(f_{E_8}) \approx 25. \tag{40}$$

We also find

$$4\pi\mathrm{Re}(f_{E_8'}) \approx \frac{n_2 b}{2\pi n_1}\ln(M_{GUT}^3/|W_F|) \approx \frac{n_2 b}{n_1}[4.4 + \frac{1}{2\pi}\ln(\mathrm{TeV}/m_{3/2})],$$

using $M_{GUT} \approx 2\pi R_6^{-1} \approx 2\times 10^{17}$ GeV, and also the expression of W_F in (35) together with the assumption that supersymmetry breaking is mainly due to W_F, not due to the M-theoretic non-perturbative term W_M in (34), and thus $W_F \approx m_{3/2}M_P^2$. (This assumption is not so crucial for our discussion.) Combining these with (11), we obtain

$$\frac{1}{2}\sum(l_I - l_I')\mathrm{Re}(T_I) \approx 25 - \frac{n_2 b}{n_1}[4.4 + \frac{1}{2\pi}\ln(\mathrm{TeV}/m_{3/2})] - \mathrm{Re}(\triangle_{E_8} - \triangle_{E_8'}), \tag{41}$$

where $\frac{1}{2}(l_I - l_I')$ are integral as required by (10).

In order to have a $U(1)_{PQ}$-symmetry which avoids the hidden sector anomaly, one needs at least one of $\frac{1}{2}(l_I - l'_I)$ to be a nonzero integer. Furthermore, as we have noticed, $\triangle_a$ in the gauge kinetic function corresponds to higher order correction in the strong coupling expansion and thus $\mathrm{Re}(\triangle_a)$ is essentially of order one or less in the M-theory limit with $\mathrm{Re}(T_I) \gg 1$ [8]. Let us also recall that b is the positive coefficient of one-loop beta function, and n_1 and n_2 are model-dependent positive integers. Then comparing the large radius condition (39) with the phenomenological relation (41), we easily find that they can be compatible only for a rather limited set of the coefficients $\{\frac{1}{2}(l_I - l'_I)\}$. For instance, if anyone of $\frac{1}{2}(l_I - l'_I)$ is significantly bigger than one, there has to be another coefficient with a similar magnitude but with a different sign.

In the above, we have noted that the phenomenological viability of the large radius condition (39) crucially depends upon the quantized coefficients of the Kähler moduli superfields T_I in gauge kinetic functions, and in fact it is viable only for a rather restricted class of models. This is particularly true for supersymmetry-preserving compactifications on a smooth Calabi-Yau manifold with vanishing E'_8 field strength. The hidden sector of such compactifications does not contain any matter multiplet, and then $n_2 b/n_1$ corresponds to the second Casimir $C_2 = \mathrm{tr}(T^2_{\mathrm{adj}})$ of the gauge group giving a dominant contribution to the field-theoretic nonperturbative superpotential W_F through the gaugino condensation. Since the hidden sector gauge group G_h is a subgroup of E'_8 commuting with the Wilson lines in the model, we have $2 \leq C_2 \leq 30$. (If $G_h = \prod SU(N_i) \times \prod U(1)$ for instance, $C_2 = \mathrm{Max}(N_i)$.) In sect. III, we have shown that $l_I = -l'_I$ in generic compactifications on a smooth six-manifold. It is also noted that if $F'_{AB} = 0$, then one of the coefficients $\frac{1}{2}(l_I - l'_I) = l_I$ is a positive integer, while the other coefficients are still non-negative (see Eq. (15)). Summarizing these, for supersymmetry-preserving compactifications on a smooth Calabi-Yau manifold with $F'_{AB} = 0$, we have

$$\sum l_I \mathrm{Re}(T_I) \approx 25 - C_2[4.4 + \frac{1}{2\pi} \ln(\mathrm{TeV}/m_{3/2})] - \mathrm{Re}(\triangle_{E_8} - \triangle_{E'_8}), \qquad (42)$$

where at least one of the non-negative integers $l_I = \int \omega^I \wedge I_4$ is non-zero, and $2 \leq C_2 \leq 30$.

Since $\mathrm{Re}(\triangle_a)$ corresponds to a subleading part of order one or less, the above result implies that (i) the nonvanishing l_I has to be fixed to be one, and (ii) C_2 can *not* be significantly bigger than its minimal value two, in order for the large radius condition (39) to be satisfied. In view of the boundary condition $\frac{1}{\pi}[G]_{E_8} = I_4$ (see the discussion below Eq. (14)), this is possible only when (i) the quantized flux of the antisymmetric tensor field in M-theory has the minimal nonzero value: $[G/2\pi] = 1/2$ in the notation of ref. [14], and (ii) the hidden gauge group E_8' is broken by Wilson lines to a subgroup with small values of the second Casimir $C_2 = \mathrm{tr}(T^2_{\mathrm{adj}})$.

As is well known, the QCD axion with a decay constant $v \gg 10^{12}$ GeV can be cosmologically troublesome [32]. Let us assume that axionic strings were inflated away in the early stage, and thus ignore the relic axions emitted from axionic strings. However still the coherent axion oscillation after the QCD phase transition in the early universe gives rise to relic axions which may overclose the universe at the present [15]. If there is no entropy production after the QCD phase transition, the relic axion mass density (in the unit of the critical energy density) at the present is given by [33]

$$\Omega_a \approx \left(\frac{\delta\theta}{3\times 10^{-3}}\right)^2 \left(\frac{v}{10^{16}\,\mathrm{GeV}}\right)^{1.18} \left(\frac{\Lambda_{QCD}}{200\,\mathrm{MeV}}\right)^{-0.7}, \tag{43}$$

where $\delta\theta = \delta a/v$ denotes the misalignment angle of the axion field at the time of QCD phase transition in the early universe. For the case with a late-time entropy production, the relic axions are diluted as [17]

$$\Omega_a \approx \delta\theta^2 \left(\frac{v}{10^{16}\,\mathrm{GeV}}\right)^{1.5} \left(\frac{T_{RH}}{6\,\mathrm{MeV}}\right)^2 \left(\frac{\Lambda_{QCD}}{200\,\mathrm{MeV}}\right)^{-2}, \tag{44}$$

where the big-bang nucleosynthesis requires the reheat temperature $T_{RH} \gtrsim 6$ MeV.

The above formulae for the relic axion energy density indicates that if $v \gg 10^{12}$ GeV, one needs either a mechanism for the axion misalignment $\delta\theta \ll 1$, or a late time entropy production, or both. They also imply that $v \gg 10^{16}$ GeV should be distinguished from $v \lesssim 10^{16}$ GeV. In order to be cosmologically viable, the former requires a significant suppression of $\delta\theta$ independently of whether there is a late time entropy production or not, while the latter can be viable only with a late time entropy production with $T_{RH} \gtrsim 6$ MeV.

The late time entropy production has been suggested a long time ago as a mechanism to make $v \gg 10^{12}$ GeV cosmologically viable [16]. In fact, it can occur naturally in string effective supergravity models [17,34]. For instance, moduli with $m = O(10)$ TeV or the flaton fields triggering thermal inflation lead to a huge entropy production after the QCD phase transition but still before the big-bang nucleosynthesis [19,35]. It was argued also that $\delta\theta$ may be relaxed down to a small value if there is a period in the early universe during which the expectation values of some moduli fields differ from the present ones and as a consequence a large effective axion mass $m_{\rm eff}$ is induced [18,19]. If $m_{\rm eff}$ is bigger than the Hubble expansion rate H, the axion field would be driven to the vacuum value in this period. However usually the moduli values to raise up the axion mass raise up also the vacuum energy density [36], and thereby it is difficult to arrange $m_{\rm eff} \gtrsim H$. Furthermore, the axion vacuum value in this period generically differs from the present one by the order of v, which would result in $\delta\theta \approx 1$, *unless* the expectation values of the moduli which affect CP violating phases are (approximately) same as the present values. Due to these difficulties, the mechanism for $\delta\theta \ll 1$ appears to involve too many cosmological assumptions.

It is rather obvious that $v \gg 10^{12}$ GeV for the QCD axion in M-theory limit. However in view of the above discussion, it is still a relevant question to ask whether $v \gg 10^{16}$ GeV or $v \lesssim 10^{16}$ GeV. To answer this question, let us estimate the QCD axion decay constant in a simple model with $h_{1,1} = 1$. It has been pointed out recently that a simple dimensional reduction of eleven-dimensional supergravity leads to the Kähler potential of S and T which is very similar to the one obtained in weakly coupled heterotic string theory [37]:

$$K = -\ln(S + S^*) - 3\ln(T + T^*).$$

This Kähler potential may receive a sizable M-theoretical correction, however it is expected that our follwing analysis is not significantly affected by this correction as long as $\mathrm{Re}(S) \gg 1$ and $\mathrm{Re}(T) \gg 1$. Using the above Kähler potential and the gauge kinetic functions in (26) with $l = 1$, we find

$$\mathcal{L}_{\rm axion} = \frac{1}{2}(\partial_\mu a)^2 + \frac{1}{2}(\partial_\mu a')^2 + \frac{1}{32\pi^2}\frac{a}{v}F^{\mu\nu}\tilde{F}_{\mu\nu} + \frac{1}{32\pi^2}\frac{a'}{v'}F'^{\mu\nu}\tilde{F}'_{\mu\nu} + ...,$$

where the ellipsis denotes the irrelevant terms including the high energy potential $\delta V_{\rm axion}$ which has been argued to be of order $e^{-2\pi T}m_{3/2}^2 M_P^2$ in the previous section, and the axion fields a and a' are defined as

$$a = \frac{2\pi(v_S^2 {\rm Im}(S) + v_T^2 {\rm Im}(T)/2)}{\sqrt{v_S^2 + v_T^2}}, \quad a' = \frac{2\pi v_S v_T ({\rm Im}(S) - {\rm Im}(T)/2)}{\sqrt{v_S^2 + v_T^2}},$$

for the axion scales

$$v_S = \frac{M_P}{2\pi\sqrt{2}\langle {\rm Re}(S)\rangle}, \quad v_T = \frac{\sqrt{3}M_P}{\pi\sqrt{2}\langle {\rm Re}(T)\rangle}. \tag{45}$$

Here the axion decay constants v and v' are given by

$$v = \frac{1}{2}\sqrt{v_S^2 + v_T^2}, \quad v' = \frac{v_S v_T}{\sqrt{v_S^2 + v_T^2}}. \tag{46}$$

The field combination a corresponds to the QCD axion which would solve the strong CP problem if ${\rm Re}(T)$ satisfies the large radius condition (39), while the other combination a' couples to the hidden sector anomaly and thus irrelvant for the strong CP problem. Here we assumed that the axion potential due to the hidden sector anomaly is much bigger than the QCD induced potential $V_{QCD} \approx f_\pi^2 m_\pi^2$, and thus $m_{a'} \gg m_a$. Note that in models with multi-gaugino (matter) condensations which are comparable to each other, the axion potential due to the hidden sector anomaly is of order $m_{3/2}^2 M_P^2$, and then the hidden sector axion a' receives a mass of order $m_{3/2}M_P/v' \approx 10^2 m_{3/2}$.

In M-theory limit, we have roughly ${\rm Re}(S) \approx {\rm Re}(T) \approx 1/\alpha_{GUT}$. Let us recall that the phenomenological relations (40) and (41) suggest that neither ${\rm Re}(S)$ nor ${\rm Re}(T)$ can be significantly bigger than $1/\alpha_{GUT}$. We then find from (45) and (46) that the QCD axion decay constant in M-theory limit is given by

$$v \approx 5\times 10^{15}\sqrt{(25/{\rm Re}(S))^2 + (85/{\rm Re}(T))^2}\,{\rm GeV} \approx 2\times 10^{16}\, GeV. \tag{47}$$

Here we stress that the large radius with ${\rm Re}(T) \approx 1/\alpha_{GUT}$ is crucial for $v \approx 10^{16}$ GeV, *not* the order of M_P, so that the QCD axion can be cosmologically viable in the presence of a late time entropy production without assuming any significant suppression of the axion

misalignment angle. If $\mathrm{Re}(T) \ll 1/\alpha_{GUT}$, the axion scale $v_{_T}$ of the model-dependent Kähler axion would be of order M_P although the axion scale $v_{_S}$ of the model-independent axion is still smaller than M_P by two orders of magnitudes: $v_{_S} \approx \frac{\alpha_{GUT}}{4\pi} M_P \approx 10^{16}$ GeV. In this case, the QCD axion decay constant v is essentially given by $v_{_T} \gg v_{_S}$, and as a result it would be of order M_P if $\mathrm{Re}(T) \lesssim 1$ for instance.

The energy density crisis of the QCD axion in M-theory may be ameliorated if there is an *accidental* axion-like field with a smaller decay constant [2]. Let a_1 denote the string-theoretic QCD axion which corresponds to a linear combination of the model-independent axion $\mathrm{Im}(S)$ and the model-dependent Kähler axions $\mathrm{Im}(T_I)$. As discussed above, the decay constant and the high energy potential of a_1 obey $v_1 \approx 10^{16}$ GeV and $\delta V_{a_1} \approx e^{-2\pi T} m_{3/2}^2 M_P^2 \lesssim 10^{-9} f_\pi^2 m_\pi^2$ for $\mathrm{Re}(T) \approx 1/\alpha_{GUT}$. Let us suppose that there is an *accidental* axion-like field a_2 with an unspecified high energy potential δV_{a_2} and the decay constant $v_2 \ll v_1$. The axion effective lagrangian is then given by:

$$\mathcal{L}_{\rm axion} = \frac{1}{2}(\partial_\mu a_1)^2 + \frac{1}{2}(\partial_\mu a_2)^2 + \frac{1}{32\pi^2}(\frac{a_1}{v_1} + \frac{a_2}{v_2})F^{\mu\nu}\tilde{F}_{\mu\nu} - \delta V_{a_2},$$

where we have ignored the high energy potential of a_1.

In this case, we have two pseudo-Goldstone bosons whose relic mass densities would be determined by the mass eigenvalues m_i and the corresponding axion misalignments δa_i $(i = 1, 2)$. Let us examine how the presence of the accidental axion a_2 affects the cosmology of the string-theoretic QCD axion a_1. If $\delta V_{a_2} \gtrsim f_\pi^2 m_\pi^2$, we have

$$(m_1 \approx \frac{f_\pi m_\pi}{v_1}, \quad \delta a_1 \approx v_1), \quad (m_2 \approx \frac{\sqrt{\delta V_{a_2}}}{v_2}, \quad \delta a_2 \approx v_2).$$

Obviously then having a_2 with $v_2 \ll v_1$ changes neither the mass nor the initial misalignment of the original axion a_1, and as a result our previous discussion of the relic mass density of the string-theoretic QCD axion remains to be valid. Furthermore, in this case the accidental axion-like field a_2 can lead to its own cosmological problem *unless* m_2 is large enough (or v_2 is small enough).

In the case with $\delta V_{a_2} \ll f_\pi^2 m_\pi^2$, the mass eigenvalues and the misalignments are given by

$$(m_1 \approx \frac{\sqrt{\delta V_{a_2}}}{v_1}, \quad \delta a_1 \approx v_1), \quad (m_2 \approx \frac{f_\pi m_\pi}{v_2}, \quad \delta a_2 \approx v_2).$$

Then a_2 corresponds to the true QCD axion with a decay constant $v_2 \ll v_1 \approx 10^{16}$ GeV, which would be cosmologically safe without any late time entropy production or any suppression of the misalignment angle $\delta\theta_2 = \delta a_2/v_2$ if $v_2 \lesssim 10^{12}$ GeV. However the other pseudo-Goldstone boson which is mainly the string-theoretic axion a_1 can still be cosmologically troublesome. If there is no entropy production below the temperature $T_{\rm osc} \approx \sqrt{m_1 M_P}$, the relic mass density of a_1 (again in the unit of the critical density) at the present would be given by

$$\Omega_{a_1} \approx 5 \times 10^3 \left(\frac{\delta V_{a_2}}{f_\pi^2 m_\pi^2}\right)^{1/4} \left(\frac{v_1}{10^{16}{\rm GeV}}\right)^{1.5} \left(\frac{\delta a_1}{v_1}\right)^2 .$$

Although there is a suppression of the relic mass density by the small factor $(\delta V_{a_2}/f_\pi^2 m_\pi^2)^{1/4}$, this would not be so significant *unless* $\delta V_{a_2}/f_\pi^2 m_\pi^2$ is extremely small. For instance, in the absence of a late time entropy production after $T_{\rm osc} \approx \sqrt{m_1 M_P}$ and also of any suppression of the misalignment angle $\delta\theta_1 = \delta a_1/v_1$, the pseudo-Goldstone boson a_1 would be cosmologically safe only when $\delta V_{a_2} \lesssim 10^{-15} f_\pi m_\pi^2$. In view of the absence of an exact continuous global symmetry in string theory [38], such an extremely small high energy potential of the accidental axion-like field is highly unlikely. We thus conclude that having an accidental axion like field with a decay constant much smaller than 10^{16} is not so helpful for ameliorating the cosmological difficulty of the QCD axion in M-theory whose decay constant was estimated to be of order 10^{16} GeV.

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